Lesson 12: The Graph of the Equation

Classwork

In Module 1, you graphed equations such as by plotting the points on the Cartesian coordinate plane that corresponded to all of the ordered pairs of numbers that were in the solution set. We called the geometric figure that resulted from plotting those points in the plane the *graph of the equation in two variables*.

In this lesson, we extend this notion of the graph of an equation to the graph of for a function . In doing so, we use computer thought code to describe the process of generating the ordered pairs in the graph of .

**Example 1**

In the previous lesson, we studied a simple type of instruction that computers perform called a for-next loop. Another simple type of instruction is an *if-then statement*. Below is example code of a program that tests for and prints “True” when ; otherwise it prints “False.”

|  |
| --- |
| **Declare integer For all from 1 to 4  If then  Print True  else  Print False  End if Next** |

The output of this program code is

False  
True  
False  
False

Notice that the if-then statement in the code above is really just testing whether each number in the loop is in the solution set.

**Example 2**

Perform the instructions in the following programming code as if you were a computer and your paper were the computer screen.

|  |
| --- |
| **Declare integer Initialize as {} For all from 0 to 4  If then  Append to   else  Do NOT append to   End if Next  Print** |

Output:

Discussion

Compare the for-next/if-then code above to the following set-builder notation we used to describe solution sets in Module 1:

Check to see that the set-builder notation also generates the set . *Whenever you see set-builder notation to describe a set, a powerful way to interpret that notation is to think of the set as being generated by a program like the for-next or if-then code above.*

Exploratory Challenge 1

Next we write code that generates a graph of a *two-variable equation* for in   
and in . The solution set of this equation is generated by testing each ordered pair in the set,

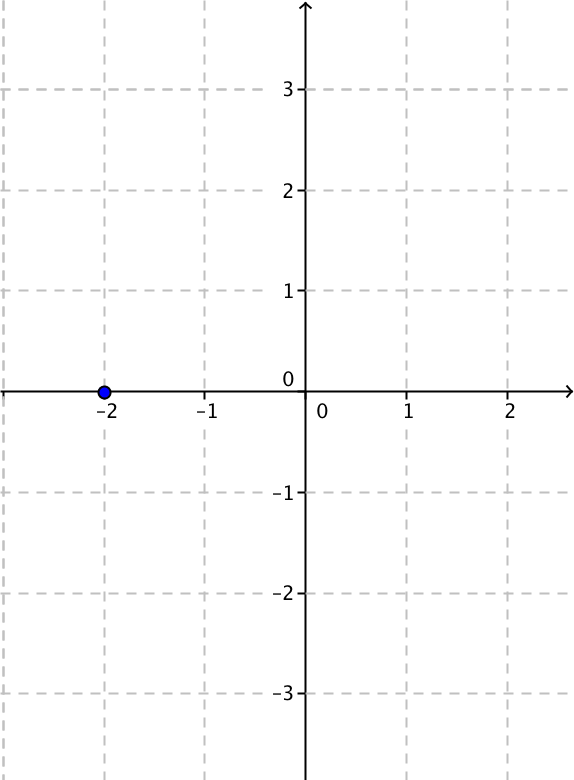
to see if it is a solution to the equation. Then the graph is just the plot of solutions in the Cartesian plane. We can instruct a computer to find these points and plot them using the following program.

|  |  |  |
| --- | --- | --- |
| **Declare and integers Initialize as {} For all in   For all in   If then  Append to   else  Do NOT append to   End if  Next  Next  Print  Plot** |  | **Loops through each for Tests whether , then for , is a solution. and so on (see arrows in  table below).** |

* 1. Use the table below to record the decisions a computer would make when following the program instructions above. Fill in each cell with “Yes” or “No” depending on whether the ordered pair would be appended or not. (The step where has been done for you.)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
|  | No |  |  |  |  |
|  | Yes |  |  |  |  |
|  | No |  |  |  |  |

* 1. What would be the output to the Print command? (The first ordered pair is listed for you.)  
       
     Output:  
     { , \_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_ }
  2. Plot the solution set in the Cartesian plane. (The first ordered pair in has been plotted for you.)



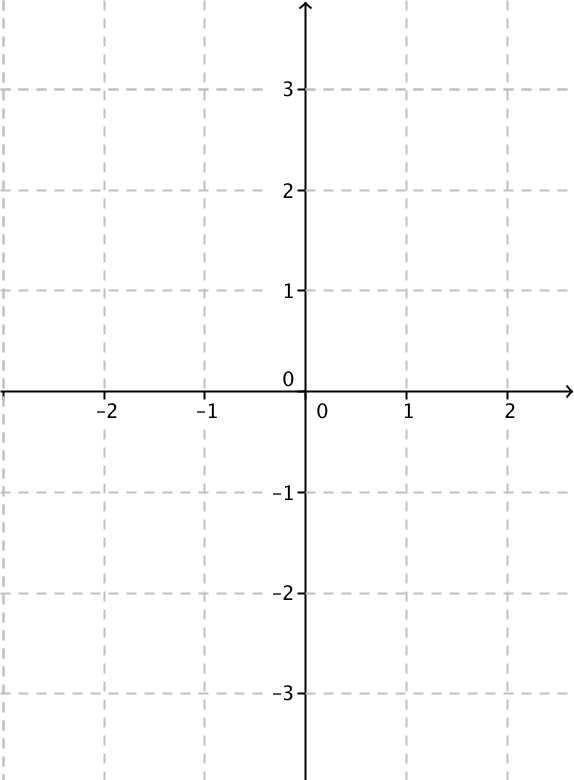
Exploratory Challenge 2

The program code in Exercise 3 is a way to imagine how set-builder notation generates solution sets and figures in the plane. Given a function with domain and range all real numbers, a slight modification of the program code above can be used to generate the graph of the equation :

Even though the code below cannot be run on a computer, students can run the following thought code in their minds.

|  |  |  |
| --- | --- | --- |
| **Declare and real Let  Initialize as {} For all in the real numbers  For all in the real numbers  If then  Append to   else  Do NOT append to   End if  Next  Next  Plot** |  | **For each -value, the code Tests whether loops through all -values. is a solution to .** |

* 1. Plot on the Cartesian plane (the figure drawn is called the graph of ).



* 1. Describe how the thought code is similar to the set-builder notation .
  2. A *relative maximum* for the function occurs at the -coordinate of . Substitute this point into the equation to check that it is a solution to , and then plot the point on your graph.
  3. A *relative minimum* for the function occurs at the -coordinate of . A similar calculation as you did above shows that this point is also a solution to . Plot this point on your graph.
  4. Look at your graph. On what interval(s) is the function decreasing?
  5. Look at your graph. On what interval(s) is the function increasing?

Lesson Summary

* **Graph of :**  Given a function whose domain , and the range are subsets of the real numbers, the *graph of*  is the set of ordered pairs in the Cartesian plane given by

When we write for the graph of , it is understood that the domain is the largest set of real numbers for which the function is defined.

* The graph of is the same as the graph of the equation .
* **Increasing/Decreasing:**  Given a function whose domain and range are subsets of the real numbers,   
  and is an interval contained within the domain, the function is called *increasing on the interval* if

whenever in .

It is called *decreasing on the interval* if

whenever in .

Problem Set

1. Perform the instructions in the following programming code as if you were a computer and your paper were the computer screen.

|  |
| --- |
| **Declare integer For all from 1 to 6  If then  Print True  else  Print False  End if Next** |

1. Answer the following questions about the computer programming code.

|  |
| --- |
| **Declare integer Initialize as {} For all from 3 to 3  If then  Append to   else  Do NOT append to   End if Next  Print** |

* 1. Perform the instructions in the programming code as if you were a computer and your paper were the computer screen.
  2. Write a description of the set using set-builder notation.

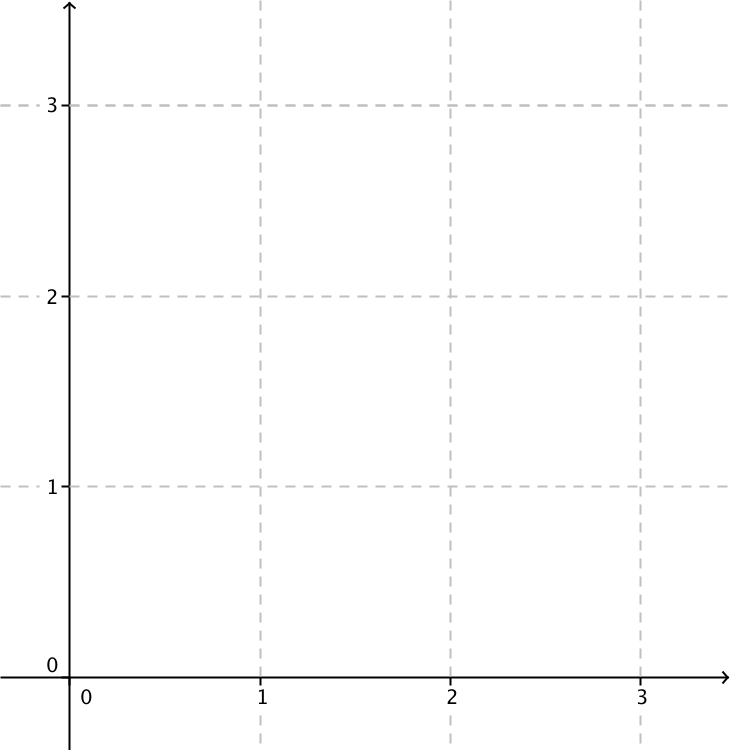
1. Answer the following questions about the computer programming code.

|  |
| --- |
| **Declare and integers Initialize as {} For all in   For all in   If then  Append to   else  Do NOT append to   End if  Next  Next  Plot** |

* 1. Use the table below to record the decisions a computer would make when following the program instructions above. Fill in each cell with “Yes” or “No” depending on whether the ordered pair would be appended or not.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

* 1. Plot the set in the Cartesian plane.



1. Answer the following questions about the thought code.

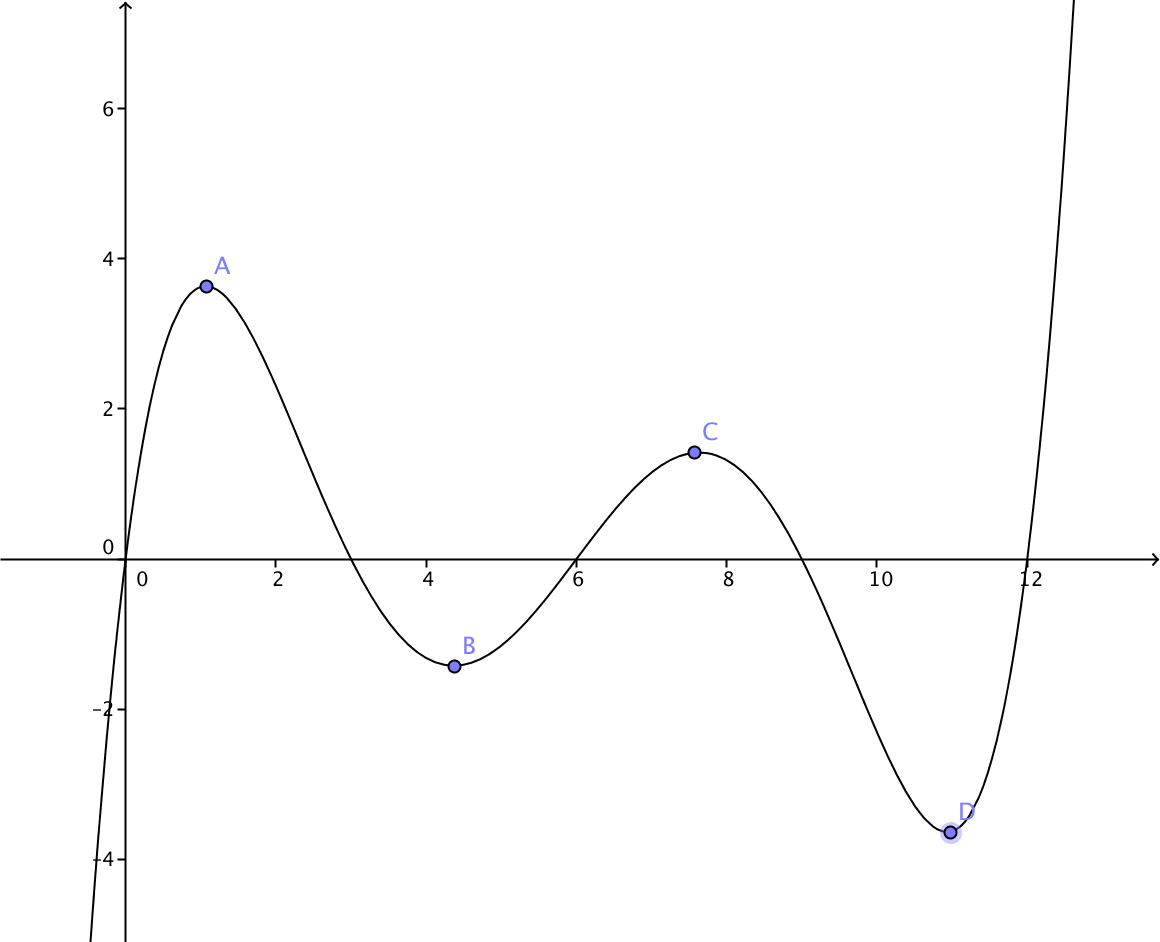
|  |
| --- |
| **Declare and real Let  Initialize as {} For all in the real numbers  For all in the real numbers  If then  Append to   else  Do NOT append to   End if  Next  Next  Plot** |

* 1. What is the domain of the function ?
  2. What is the range of the function ?
  3. Write the set generated by the thought code in set-builder notation.
  4. Plot the set to obtain the graph of the function .
  5. The function is clearly a decreasing function on the domain of the real numbers. Show that the function satisfies the definition of decreasing for the points and on the number line; that is, show that since , then .

1. Sketch the graph of the functions defined by the following formulas, and write the graph of as a set using set-builder notation. (Hint: For each function below, you can assume the domain is all real numbers.)
2. Answer the following questions about the set:

.

* 1. The equation can be rewritten in the form where . What are the domain and range of the function specified by the set?
     1. Domain:
     2. Range:
  2. Write thought code such as that in Problem 4 that will generate and then plot the set.

1. Answer the following about the graph of a function below.
   1. Which points (A, B, C, or D) are relative maxima?
   2. Which points (A, B, C, or D) are relative minima?
   3. Name any interval where the function is increasing.
   4. Name any interval where the function is decreasing.